

Anomaly-Free Gauged $U(1)'$ in Local Supersymmetry and Baryon-Number Violation

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Abstract

The supersymmetric extension of the standard model suffers from a problem of baryon-number violation. Discrete (and global) symmetries introduced to protect the proton are unstable under gravitational effects. We add a gauged $U(1)_X$ to the standard model gauge group G_{SM} and require it to be anomaly-free. As new (chiral) superfields we only allow G_{SM} -singlets in order to maintain the good unification predictions. We find the most general set of solutions for the rational singlet charges. We embed our models in *local* supersymmetry and study the breaking of supersymmetry and $U(1)_X$ to determine M_X . We determine the full non-renormalizable and gauge invariant Lagrangian for the different solutions. We expect any effective theory to contain baryon- and lepton-number violating terms of dimension four suppressed by powers of M_X/M_{Pl} . The power is predicted by the $U(1)_X$ charges. We find consistency with the experimental bounds on the proton lifetime and on the neutrino masses. We also expect all supersymmetric models to have an unstable but longlived lightest supersymmetric particle. Consistency with underground experiments on upward going muons leads to stricter constraints than the proton decay experiments. These are barely satisfied.

1 Introduction

When incorporating supersymmetry into the Standard Model one immediately runs into a problem. The Standard Model conserves baryon- (B) and lepton-number (L) automatically and higher dimensional $\Delta B, \Delta L \neq 0$ operators are suppressed by at least 4 powers of the scale of baryon- or lepton-number violation. However, in supersymmetry [1] the most general interactions involving the Standard Model (super-) fields and invariant under the Standard Model gauge group

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \quad (1)$$

include $\Delta B, \Delta L \neq 0$ terms of dimension 4 [2, 3, 4]

$$(LL\bar{E})_F, \quad (LQ\bar{D})_F, \quad (\bar{U}\bar{D}\bar{D})_F, \quad (L\bar{H})_F. \quad (2)$$

Here L, Q are the left-handed lepton and quark superfields respectively.¹ $\bar{E}, \bar{U}, \bar{D}$ are the corresponding right-handed superfields. Their G_{SM} quantum numbers are given below in Eq.(6). There are also dim 5 terms which can lead to dangerous levels of proton decay [1, 2]

$$\begin{aligned} & (LL\bar{H}\bar{H})_F, \quad (QQQL)_F, \quad (\bar{U}\bar{U}\bar{D}\bar{E})_F, \quad (LH\bar{H}\bar{H})_F, \\ & (QQQH)_F, \quad (HQ\bar{U}\bar{E})_F, \quad (Q\bar{U}L^*)_D, \quad (\bar{U}\bar{D}^*\bar{E})_D, \\ & (\bar{H}\bar{H}\bar{E}^*)_D, \quad (\bar{H}^*H\bar{E})_D. \end{aligned} \quad (3)$$

With the gauge structure restricted to that of the minimal supersymmetric standard model (MSSM) the $L\bar{H}$ term in Eq.(2) can be rotated away through a field redefinition. It is then absorbed in the $LL\bar{E}$ and $LQ\bar{D}$ terms as well as those of the MSSM. However, if extra gauge symmetries are present which distinguish between L and H this is no longer true. This gauge symmetry could prohibit the first two terms of Eq.(2) as well as the $\mu H\bar{H}$ term of the MSSM while allowing a $\kappa L\bar{H}$ term. This can then no longer be rotated away. In the low-energy effective theory this extra symmetry is broken. If it is broken at sufficiently high energy no effects are observable and the $L\bar{H}$ term can again be rotated away. We thus retain the $L\bar{H}$ term in our discussion and decide if there is a remnant effective $L\bar{H}$ term.

The dim 4-terms together lead to an unacceptable level of proton decay. Thus the symmetry of the SM must be extended to

$$G = G_{SM} \otimes \tilde{G}, \quad (4)$$

such that \tilde{G} guarantees the longevity of the proton. Discrete, global, and gauge symmetries have been considered. The first solution to the problem of proton decay was to introduce the discrete symmetry R-parity [5, 3, 4] which prohibits all the terms in (2). Later other discrete symmetries were considered, in particular baryon-parity [6] which only prohibits the term $\bar{U}\bar{D}\bar{D}$. This is sufficient to protect the proton. However, Krauss and Wilczek [7] later showed that discrete symmetries are violated by gravitational effects unless they are remnants of a broken gauge symmetry. So only very special discrete symmetries, discrete gauge symmetries, are viable candidates for \tilde{G} . Ibanez and Ross [8, 6] translated the requirement of an anomaly-free gauge theory into a set of conditions on the remnant discrete symmetry. They then systematically studied all family-independent discrete symmetries up to \mathbf{Z}_4 . They found that only R-parity and baryon-parity are anomaly-free in this sense. Baryon-parity has the advantage of forbidding the dangerous dim-5 $\Delta B \neq 0$ operators (3). Both symmetries are consistent with all known experiments but lead to drastically different predictions for future collider searches of supersymmetry [9, 10].

¹We have used the same symbol L for lepton number and for the left-handed lepton doublet. The context should always make it clear which is meant.

Global symmetries suffer from the same gravitational breaking as discrete symmetries. However, they can not be the remnant of an anomaly-free broken gauge symmetry. In addition a global baryon number for example leads to problems with baryogenesis and if broken a global symmetry can lead to an unwanted axion. Therefore we do not further consider global symmetries.

Thus it is most likely that the proton is protected by a (broken) gauge symmetry. A first solution to the problem of baryon-number violation via a gauge symmetry $\tilde{G} = U(1)_X$ was considered by S. Weinberg [2]. His approach was to give all the matter fields charges Q_X of the same sign and the Higgs fields the opposite sign.² In order to guarantee an anomaly-free theory it is then necessary to include fields which transform non-trivially under G_{SM} , in particular also additional ($SU(3)$) coloured fields. Similar models have later been constructed by [12]. However, the extra coloured field must be massive and such a mass term is in general not $U(1)_X$ gauge invariant. In principle it can however get a heavy mass when $U(1)_X$ is broken.

Besides the problem of giving G_{SM} non-singlets a mass such fields also drastically affect the success of the unification of the gauge couplings in supersymmetry provided their masses are below the unification scale. We shall thus only consider G_{SM} -singlets as additional fields to cancel the $U(1)_X$ anomalies. In this case there is no longer an anomaly-free $U(1)_X$ where all matter fields have the same charge and the problem of $\Delta B, \Delta L \neq 0$ interactions must be reconsidered.

This approach was first studied by Font *et al.* [13]. The authors searched for a $U(1)_X$ which prohibits all dangerous dimension four terms (2). They found that $U(1)_X$ necessarily acts on the MSSM matter fields as a linear combination of hypercharge Y and the third component of right-handed iso-spin I_3^R . Since $(B - L)$ is a linear combination of Y and I_3^R , $U(1)_X$ can also be considered as a linear combination of Y and $(B - L)$

$$X = \alpha_Y Y + \alpha_{(B-L)}(B - L) + \alpha_S S, \quad (5)$$

here S is the part of $U(1)_X$ acting on the singlets. Later, in [8, 6] this work was extended to search for a $U(1)_X$ which only prohibits a subclass of the terms (2). They found both a gauged baryon-number and a gauged lepton-number. However, the anomaly-free solutions require additional fields (beyond those of MSSM) transforming non-trivially under G_{SM} , for example the leptoquark of E_6 . As stated above, we do not further consider this approach here.

Since in supersymmetry the proton is most likely protected by a (broken) gauge symmetry, we study the possible $U(1)_X$ solutions in detail. We extend the work of [13] in several points. First, we determine the most general $U(1)_S$ of (5) which is anomaly-free. For three additional G_{SM} singlets we find an infinite set of rational solutions. The singlet charges are generally not identical and thus an interpretation in terms of right-handed neutrinos and an embedding in (a family independent) $SO(10)$ is not necessary. Second, due to the importance of gravitational effects when considering discrete symmetries we embed our models in local supersymmetry. We consider local supersymmetry as an effective theory of a more complete unified theory including

²This is just the gauged Fayet $U(1)$ [11] which was introduced in order to give the scalar fermions large positive mass corrections after supersymmetry breaking.

gravity. We can then study the spontaneous breaking of supersymmetry and $U(1)_X$ explicitly. We determine the scale of $U(1)_X$ breaking (M_X) dynamically in terms of the Planck scale M_{Pl} and the supersymmetry scale. Before $U(1)_X$ is broken, but below M_{Pl} the superpotential can in principle contain *all non-renormalizable* terms compatible with $G_{SM} \otimes U(1)_X$ suppressed by powers of M_{Pl}^{-1} . These terms could be generated by loop effects from the broken unified theory. We consider all such terms and their possible effects at low energies, even though in any explicit (yet to be constructed and therefore unknown) unified model we expect only a subset of these terms to be generated via loop effects. After the breaking of $U(1)_X$ at M_X , the renormalizable superpotential will then *always* contain all terms (2) suppressed by powers of (M_X/M_{Pl}) . We thus predict the scale of B and L violating effects including neutrino masses in the effective theory and find them compatible with experiment. An interesting effect of this scenario is that we always find an unstable but long-lived lightest supersymmetric particle (LSP). We find the cosmological constraints on the LSP lifetime to be significantly stricter than those due to proton decay and only barely compatible with our models.

The above discussion refers to $U(1)$ gauge symmetries. In a separate publication we discuss gauged $U(1)$ R-symmetries [14].

2 Anomaly Cancellation Conditions

We shall embed our $G_{SM} \otimes U(1)_X$ models in $N = 1$ local supersymmetry. In accordance with our philosophy stated above, the matter chiral multiplets are those of the MSSM with the addition of G_{SM} singlets, N , and Z_m . These multiplets are denoted by

$$\begin{aligned}
L : (1, 2, -\tfrac{1}{2}, l), & \quad \bar{E} : (1, 1, 1, e), & Q : (3, 2, \tfrac{1}{6}, q), \\
\bar{U} : (\bar{3}, 1, -\tfrac{2}{3}, u), & \quad \bar{D} : (\bar{3}, 1, \tfrac{1}{3}, d), & H : (1, 2, -\tfrac{1}{2}, h), \\
\bar{H} : (1, \bar{2}, \tfrac{1}{2}, \bar{h}) & \quad N : (1, 1, 0, n), & Z_m : (1, 1, 0, z_m),
\end{aligned} \tag{6}$$

where we have indicated in parentheses the $G_{SM} \otimes U(1)_X$ quantum numbers. We shall assume that the superpotential in the observable sector has the form

$$g^{(O)} = h_E^{ij} L^i E^{cj} H + h_D^{ij} Q^i D^{cj} H + h_U^{ij} Q^i U^{cj} \bar{H} + h_N N H \bar{H} \tag{7}$$

where h_E, h_D, h_U, h_N are Yukawa couplings. Thus at this stage we assume the theory conserves R-parity. We have added the term $NH\bar{H}$ instead of $\mu H\bar{H}$ as in the MSSM, in order to incorporate a possible solution to the μ -problem. We shall show below that this is not possible. The singlet couplings will be determined by the charges z_m obtained from the solutions to the anomaly equations below.

To build a realistic model the superpotential should be gauge-invariant and the new gauge symmetry $U(1)_X$ should be anomaly-free, *i.e.*

$$l + e + h = 0, \tag{8}$$

$$q + d + h = 0, \tag{9}$$

$$q + u + \bar{h} = 0, \quad (10)$$

$$n + h + \bar{h} = 0, \quad (11)$$

$$3\left[\frac{1}{2}l + e + \frac{1}{6}q + \frac{4}{3}u + \frac{1}{3}d\right] + \frac{1}{2}(h + \bar{h}) = 0, \quad (12)$$

$$3[-l^2 + e^2 + q^2 - 2u^2 + d^2] - h^2 + \bar{h}^2 = 0, \quad (13)$$

$$3[2l^3 + e^3 + 6q^3 + 3u^3 + 3d^3] + 2h^3 + 2\bar{h}^3 + n^3 + \sum z_m^3 = 0, \quad (14)$$

$$3\left[\frac{1}{2}l + \frac{3}{2}q\right] + \frac{1}{2}(h + \bar{h}) = 0, \quad (15)$$

$$3\left(q + \frac{1}{2}u + \frac{1}{2}d\right) = 0, \quad (16)$$

$$3[2l + e + 6q + 3u + 3d] + 2(h + \bar{h}) + n + \sum z_m = 0. \quad (17)$$

The last six equations are the $Y^2X, YX^2, X^3, (SU(2)_L)^2X, (SU(3)_c)^2X$ and gravitational anomaly [15] equations, respectively. We have assumed that the $U(1)_X$ -charges are family independent, *e.g.* $l_1 = l_2 = l_3 = l$ and there are three generations. In solving this set of ten equations we first notice that the Eqs.(8)-(10), (12,13), (15), and (16) are independent of the singlets N, Z_m . The solution to these seven equation can be expressed in terms of two variables which we choose to be l and e

$$h = -l - e, \quad \bar{h} = l + e, \quad q = -\frac{1}{3}l, \quad u = -\frac{2}{3}l - e, \quad d = \frac{4}{3}l + e. \quad (18)$$

Inserting the values for h and \bar{h} into Eq.(11) we obtain

$$n = 0. \quad (19)$$

Since the $U(1)_X$ -number of the singlet N is zero it does not affect the anomalies and can be discounted. We thus eliminate it from our further discussion and replace $NH\bar{H}$ in $g^{(O)}$ by

$$\mu H\bar{H}. \quad (20)$$

We thus offer no new insights on the μ -problem. However, there is also no Peccei-Quinn axion in the theory. The remaining equations involving the singlet field charges are

$$3(2l + e) + \sum z_m = 0, \quad (21)$$

$$3(2l + e)^3 + \sum z_m^3 = 0. \quad (22)$$

The choice $l = -e/2$ would give the chiral multiplets in the observable sector $U(1)_X$ -numbers which are identical to the Y-numbers. Thus, non trivial solutions ($U(1)_X \neq U(1)_Y$) are only possible through the inclusion of the singlets Z_m . We also can see from the above equations that with the *minimal* field content (no singlets) charge quantization ($q_d = q_e/3$ etc.) is given as the unique solution to the anomaly equations. This is true in the SM and MSSM and independently of the gravitational anomaly equation [16, 13].

The cases of $l = -e/2$ with non-trivial singlet charges are not very interesting and we shall require $2l + e \neq 0$. For only one singlet there is no solution to Eqs.(21,22). For

two singlets there are no real solutions. The only real solution of the cubic equation in (22) has $e = -2l$. For three singlets an obvious solution is

$$z_1 = z_2 = z_3 = -(2l + e). \quad (23)$$

We shall normalize $z_1 = 1$ since the equations contain no constant term. For

$$z_2 = \frac{1}{2}(x + y), \quad z_3 = \frac{1}{2}(y - x), \quad (24)$$

we then obtain the quadratic equation in x :

$$2l + e = -\frac{1}{3}(y + 1), \quad (25)$$

$$27yx^2 + (y - 2)^2(5y + 8) = 0. \quad (26)$$

There are two classes of solutions. For $y = 2$ we must have $x = 0$; this corresponds to the charges

$$2l + e = -1, \quad z_1 = z_2 = z_3 = 1, \quad (27)$$

This is the same as (23). For $l = -1$ this corresponds to the gauged $(B - L)$ symmetry of $SO(10)$ and the three singlets are interpreted as the right-handed neutrinos of the $\mathbf{16}_{SO(10)}$.

The other class has $y \neq 2$. The condition for a real solution of Eq.(26) is $y \in [-\frac{8}{5}, 0)$. In order to have a rational solution we must have

$$\frac{5y + 8}{3y} = -q^2, \quad \text{or} \quad y = -\frac{8}{5 + 3q^2}, \quad q \in \mathbf{Q}. \quad (28)$$

Then

$$x = \pm \frac{q}{3} \left(\frac{8}{5 + 3q^2} + 2 \right). \quad (29)$$

The two signs for x correspond to the interchange $z_2 \leftrightarrow z_3$. Choosing the plus sign and solving for the singlet charges we obtain

$$z_1 = 1, \quad z_2 = \frac{(q - 1)(q^2 + q + 4)}{5 + 3q^2}, \quad z_3 = -\frac{(q + 1)(q^2 - q + 4)}{5 + 3q^2}. \quad (30)$$

The charges for the standard model fields are then determined by two free parameters l and q via

$$e = \frac{1 - q^2}{5 + 3q^2} - 2l. \quad (31)$$

We have thus obtained the complete set of anomaly-free solutions for $U(1)_X$ and three additional singlets.³ When discussing the details of models we shall mostly focus on the specific solutions

$$q = 0, \quad 2l + e = \frac{1}{5}, \quad z_1 = 1, z_2 = z_3 = -\frac{4}{5}, \quad (32)$$

$$q = 3, \quad 2l + e = -\frac{1}{4}, \quad z_1 = z_2 = 1, z_3 = -\frac{5}{4}. \quad (33)$$

³The case $2l + e = 0$ is included as $q = \pm 1$, $z_1 = 1$, $z_2 = 0$, $z_3 = -1$.

The other models lead to very similar results as we shall see. We can use these solutions to determine the coefficients α_i of (5). The additional $U(1)$'s which only act on the singlets are given by

$$U(1)_{S_1} : (z_1, z_2, z_3) = (1, -1, 0), \quad (34)$$

$$U(1)_{S_2} : (z_1, z_2, z_3) = (1, 0, -1), \quad (35)$$

and we have

$$U(1)_X = 2(l + e)U(1)_Y - (2l + e)U(1)_{(B-L)} \quad (36)$$

$$+ \frac{1}{3}(1 - 2z_2 + z_3)U(1)_{S_1} + \frac{1}{3}(z_2 - 2z_3 + 1)U(1)_{S_2}. \quad (37)$$

This contains all solutions, including (27). For the solutions (30) the α_i are given in terms of l, q

$$\begin{aligned} \alpha_Y &= \left(\frac{1 - q^2}{5 + 3q^2} - l \right), & \alpha_{(B-L)} &= -\frac{1 - q^2}{5 + 3q^2}, \\ \alpha_{S_1} &= -\frac{q^3 - q^2 + 3q - 3}{5 + 3q^2}, & \alpha_{S_2} &= \frac{q^3 + q^2 + 3q + 3}{5 + 3q^2}. \end{aligned} \quad (38)$$

Thus we have found the following result. Due to the free parameter l above, and taking the $z_1 = 1$ normalization into account, any rational linear combination of $U(1)_Y$ and $U(1)_{B-L}$ can be made anomaly-free acting on the MSSM fields with three additional G_{SM} singlets. For each linear combination of $U(1)_Y$ and $U(1)_{B-L}$ there is an infinite set of charges z_1, z_2, z_3 given by (30) which lead to an anomaly-free $G_{SM} \otimes U(1)_X$ model. These can be expressed as specific rational linear combination of $U(1)_{S_1, S_2}$. Next we dynamically break supersymmetry and $U(1)_X$.

3 Breaking of Supersymmetry and $U(1)_X$

To have a realistic model both supersymmetry and $U(1)_X$ must be broken at low energies. Since we have a locally supersymmetric theory, it is possible to break supersymmetry spontaneously. The easiest way is to utilize a hidden sector whose fields are singlets with respect to the Standard Model gauge group. Depending on whether the $U(1)_X$ -symmetry and supersymmetry are to be broken simultaneously or not, these singlets would have or not have non-trivial $U(1)_X$ -numbers.

We can break supersymmetry independently of the $U(1)_X$ by adding to the system one singlet Z with zero $U(1)_X$ -number. This clearly has no affect on the anomaly equations. Then we can take the superpotential

$$g = m^2(Z + \beta) + g'(Z_1, Z_2, Z_3) + g^{(O)}(S_i) + g''(Z_i, S_j), \quad (39)$$

where $g^{(O)}(S_i)$ is the observable sector superpotential which only depends on the SM chiral superfields S_i . $g''(Z_i, S_j)$ is a non-renormalizable part of the superpotential involving also interactions between Z_i and S_j . We will discuss it in more detail in the

next section. The first term is the Polonyi potential, where β is a constant. It will be fine-tuned so that the cosmological constant is zero.

In the following we shall study the two models (32,33) from the previous section. The other models will give similar results as long as two singlets have different sign charges. This is always the case for (30). It is not the case for the solutions (23) which correspond to $U(1)_X = -(2l+e)(B-L)$. In this case it is not possible to write a superpotential of weight zero for the singlet fields.

For the model (33) it is not possible to write a renormalizable potential as a function of Z_1, Z_2, Z_3 . The simplest function we can write is

$$g' = \kappa^6 Z_1^4 \left(\sum_{p=0}^5 a_p Z_2^{5-p} Z_3^p \right) \quad (40)$$

and we can assume the symmetry $Z_1 \leftrightarrow Z_2$ which implies that $a_p = a_{5-p}$. We have inserted the Planck scale in (40) in order to avoid introducing new scales, $\kappa = \sqrt{8\pi G_N} \approx 10^{-19} \text{ GeV}^{-1}$. For the model (32) the superpotential is given by

$$g' = \kappa^6 Z_3^5 \sum_{p=0}^4 a_p Z_1^{4-p} Z_2^p. \quad (41)$$

The two models lead to very similar results.

We now analyze the potential $V(Z, Z_1, Z_2, Z_3)$ with g' given in Eq.(40). We shall take the Kähler function to correspond to minimal kinetic energy for the scalar fields

$$\mathcal{K} = -\frac{\kappa^2}{2} (|Z|^2 + |Z_1|^2 + |Z_2|^2 + |Z_3|^2 + |Z_i|^2). \quad (42)$$

If we ignore the low-energy fields S_i , *i.e.* consider $g'', g^{(O)} \ll \langle Z \rangle^3, \langle Z_i \rangle^3$ we obtain for the potential

$$\begin{aligned} V = & \frac{1}{2} e^{\frac{1}{2}\kappa^2 |Z_a|^2} \left[\left| m^2 + \frac{\kappa^2}{2} \bar{Z} \left(m^2(Z + \beta) + \kappa^2 Z_3^4 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p \right) \right|^2 \right. \\ & + \left| \kappa^6 Z_3^4 \sum_{p=0}^4 (5-p) a_p Z_1^{4-p} Z_2^p + \frac{\kappa^2}{2} \bar{Z}_1 \left(m^2(Z + \beta) + \kappa^6 Z_3^4 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p \right) \right|^2 \\ & + \left| \kappa^6 Z_3^4 \sum_{p=1}^5 p a_p Z_1^{5-p} Z_2^{p-1} + \frac{\kappa^2}{2} \bar{Z}_2 \left(m^2(Z + \beta) + \kappa^6 Z_3^4 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p \right) \right|^2 \\ & + \left| 4\kappa^6 Z_3^3 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p + \frac{\kappa^2}{2} \bar{Z}_3 \left(m^2(Z + \beta) + \kappa^6 Z_3^4 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p \right) \right|^2 \\ & - \frac{3}{2} \kappa^2 \left| m^2(Z + \beta) + \kappa^6 Z_3^4 \sum_{p=0}^5 a_p Z_1^{5-p} Z_2^p \right|^2 \Big] \\ & + \left(\frac{\tilde{g}'^2}{8} \right) \left| |Z_1|^2 + |Z_2|^2 - \frac{5}{4} |Z_3|^2 \right|^2 \end{aligned} \quad (43)$$

\tilde{g}' is the $U(1)_X$ gauge coupling, \bar{Z} is the complex conjugate field of Z . We have assumed here that the gauge field kinetic energy term is minimal: $f_{\alpha\beta} = \delta_{\alpha\beta}$. Since the fields Z_i transform under the extra $U(1)_X$ group the corresponding D-term appears as the last term in the above potential. There are many possible minima for this potential, mainly with $\langle Z_1 \rangle \approx \langle Z_2 \rangle \approx \langle Z_3 \rangle \approx \langle Z \rangle \approx \frac{1}{\kappa}$. However, we are mainly interested in the situation where $\langle Z \rangle \approx \frac{1}{\kappa} \gg \langle Z_1 \rangle, \langle Z_2 \rangle, \langle Z_3 \rangle$ and we shall tune β and the constants a_p so that V is zero at such a minimum and positive definite, and where the condition

$$|Z_1|^2 + |Z_2|^2 \simeq \frac{5}{4}|Z_3|^2 \quad (44)$$

is satisfied. In this case we can expand around the vev $Z \approx \frac{1}{\kappa}$ and write down the effective potential as a function of Z_1, Z_2 , and Z_3 [17]

$$\begin{aligned} V = & |\hat{g}_{,i}|^2 + m_{3/2}^2 |Z_i|^2 + m_{3/2} \left[Z^i \hat{g}_{,i} + (A - 3)\hat{g} + h.c. \right] \\ & + \frac{\tilde{g}'^2}{8} \left| |Z_1|^2 + |Z_2|^2 - \frac{5}{4}|Z_3|^2 \right|^2 \end{aligned} \quad (45)$$

where \hat{g} is the same as g' up to a multiplication factor which can be absorbed in the a_p . The parameters A and $m_{3/2}$ are related to the potential g : $m_{3/2} \approx \kappa^2 \langle g \rangle$. For the Polonyi type potential $m_{3/2} \approx \kappa m^2$. It is now easy to minimize the potential (45). For simplicity we shall assume that $\langle Z_2 \rangle = 0$. Then

$$\begin{aligned} V = & \kappa^{12} \left| 16|Z_3|^6 |Z_1|^{10} + 25|Z_3|^8 |Z_1|^8 \right| + m_{3/2}^2 \left| |Z_1|^2 + |Z_3|^2 \right| \\ & + \kappa^6 A' m_{3/2} |Z_1^5 Z_3^4 + h.c| + \frac{\tilde{g}'^2}{8} \left(|Z_1|^2 - \frac{5}{4}|Z_3|^2 \right) \end{aligned} \quad (46)$$

minimizing with respect to Z_1 and Z_3 one finds that the equations imply

$$|Z_1|^2 - \frac{5}{4}|Z_3|^2 = \mathcal{O}(m_{3/2}^2) \quad (47)$$

and $\langle Z_1 \rangle$ satisfies the equation

$$x^2 + \left(\frac{5A'}{576} \right) m_{3/2} x + \frac{1}{2} \left(\frac{5}{32} \right)^2 m_{3/2}^2 = 0 \quad (48)$$

where $x = \kappa^6 \langle Z_1 \rangle^7$. The solution of this quadratic equation gives

$$Z_1 \approx \mathcal{O} \left(\frac{m_{3/2}}{100 \kappa^6} \right)^{\frac{1}{7}} \quad (49)$$

$$\simeq 10^{16} \text{ GeV} \equiv M_X \quad (50)$$

From the above analysis we deduce that for the solution with Z_1, Z_2 symmetric this gives

$$\langle Z_1 \rangle^2 = \langle Z_2 \rangle^2 = \frac{5}{8} |Z_3|^2 + \mathcal{O}(m_{3/2}^2) \quad (51)$$

$$= \mathcal{O}(M_X^2) \quad (52)$$

so the mass for the $U(1)$ gauge bosons is $\mathcal{O}(M_X)$. The approximation we used to obtain this solution is valid because shifts in the low-energy sector due to the breaking of the $U(1)_X$ symmetry are of order

$$\kappa^2 g' \approx \mathcal{O}(\kappa^8 M_X^9) = \mathcal{O}(1 \text{ GeV}) \quad (53)$$

For the solution (23) there is no superpotential and the above mechanism will not apply. It is still possible to break supersymmetry and $U(1)_X$ by taking a non-minimal kinetic energy of the form

$$\sum_{i=1}^3 (|Z_i|^2 + \alpha |Z_i|^4) - \ln \sum_{i=1}^3 |Z_i|^2. \quad (54)$$

However, this would only lead to solutions $\langle Z_1 \rangle = \langle Z_2 \rangle = \langle Z_3 \rangle = \mathcal{O}(\frac{1}{\kappa})$. As will be clear from the discussion in the following section these solutions will lead to terms in the effective potential which give rise to too fast proton decay since $M_X \kappa = \mathcal{O}(1)$. It does not seem that one can construct a realistic model in this case.

Since the $U(1)_X$ -symmetry is broken, there will be $U(1)_X$ -breaking terms in the effective action, and these will be induced through the $U(1)_X$ -gauge and gaugino interactions. Such terms can lead to proton decay, or to baryon and lepton number non-conserving processes. We discuss them in the next section.

4 Baryon- and Lepton-Number Violation

The baryon- and lepton-number violating terms of Eqs.(2,3) can lead to proton decay. We first discuss the dimension four terms (2). Given our solutions from Section 2 their charges Q_X can be expressed in terms of l, e ,

$$Q_X(LL\bar{E}, LQ\bar{D}, \bar{U}\bar{D}\bar{D}, L\bar{H}) = 2l + e. \quad (55)$$

For a non-trivial $U(1)_X$ we had required $2l + e \neq 0$ and thus at tree-level all dimension four terms are excluded. However, local supersymmetry is a non-renormalizable theory. We thus expect our model to be the effective action of a more complete unified model at a higher scale. We expect this unified theory to include $\Delta B, \Delta L \neq 0$ effects. The non-renormalizable terms in our model below the Planck scale will then be obtained as the 1-loop effective action of this unified theory. However, for lack of knowledge of the unified model we shall consider the most general non-renormalizable interactions between the singlets Z_i and the MSSM fields S_i which are $G_{SM} \otimes U(1)_X$ gauge-invariant

$$\kappa^N Z_1^{n_1} Z_2^{n_2} Z_3^{n_3} \prod_{i=1}^3 S_i \quad (56)$$

where $N = n_1 + n_2 + n_3$ and the S_i product is a trilinear term of Eq.(2).⁴ After breaking $U(1)_X$ in principle one has to find the one-loop effective action below M , which will

⁴Since the low-energy effective Lagrangian always contains the term $H\bar{H}$, and the terms $L\bar{H}, LL\bar{E}, LQ\bar{D}$ are either present or absent simultaneously, it is possible in our models to rotate away the $L\bar{H}$ terms and we do not further consider them. However, we emphasize that for other models this must always be checked in each case.

include $\Delta B, \Delta L \neq 0$ terms. As our singlet interactions are non-renormalizable from the start, a calculation of the effective interaction is not very reliable. But all terms in the effective action are necessarily derivable from non-renormalizable terms of the form (56). We shall then consider the terms obtained from (56) by replacing $Z_i \rightarrow \langle Z_i \rangle$. If all the $\Delta B, \Delta L \neq 0$ effects generated in this manner are suppressed, then we can conclude that $\Delta B, \Delta L \neq 0$ effects are not observable in the laboratory.

When replacing $Z_i \rightarrow \langle Z_i \rangle$ we obtain an effective superpotential containing the terms

$$(\kappa M_X)^N \prod_{i=1}^3 S_i \quad (57)$$

with effective coupling constants $\epsilon^N = (\kappa M_X)^N \ll 1$. Here $M_X \approx \langle Z_i \rangle \approx 10^{16} \text{ GeV}$ is the scale of $U(1)_X$ breaking. Since the terms $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ have the same charge Q_X they will be suppressed by the same power of $\lambda \sim \epsilon^N$. If we have an effective superpotential

$$g^{eff}(S_i) = \lambda L\bar{L}\bar{E} + \lambda' LQ\bar{D} + \lambda'' \bar{U}\bar{D}\bar{D} \quad (58)$$

then we can use previous proton decay rate calculations [18] and the experimental lower bound on the proton lifetime [19] to estimate the extremely strict bound

$$\lambda \cdot \lambda' < 10^{-27} \left(\frac{m_{squark}}{200 \text{ GeV}} \right)^2. \quad (59)$$

Since $\epsilon \approx 10^{-3}$ we must have $N \geq 5$. In our first model $(z_1, z_2, z_3) = (1, -\frac{4}{5}, -\frac{4}{5})$, $2l + e = \frac{1}{5}$ and the lowest dimensional term is

$$\kappa^7 Z_1^3 (Z_2 Z_3)^2 g^{eff}(S_i), \quad (60)$$

and indeed we get the appropriate suppression with $N = 7$. For our second model, $(z_1, z_2, z_3) = (1, 1, -\frac{5}{4})$, $2l + e = -\frac{1}{4}$ and the lowest dimensional term is

$$\kappa^7 Z_1^3 Z_2^4 g^{eff}(S_i), \quad (61)$$

and again $N = 7$. The general case for rational q is not solvable. However, we can determine an upper bound on the suppression. For terms symmetric in Z_2, Z_3 we obtain

$$\kappa^{2n_1+n_3} (Z_1 Z_2)^{n_1} Z_3^{n_3} g^{eff}(S_i) \quad (62)$$

which has the charge

$$Q = \frac{-8n_1}{5+3q^2} + n_3 + \frac{1-q^2}{5+3q^2} \equiv 0 \quad (63)$$

and must vanish. For $q \neq 1$ the lowest dimensional solutions are $n_1 = 3, n_3 = 1$ and $n_1 = 2, n_3 = 3$, and thus $N = 7$ once again. However, it is possible that for certain values of q the suppression might be weaker.

We now turn to the dimension five terms. Their charges are given by

$$Q_X(LH\bar{H}\bar{H}, \bar{H}\bar{H}\bar{E}^*) = \frac{1}{2} Q_X(LL\bar{H}\bar{H}) = 2l + e \quad (64)$$

$$Q_X(QQQH, HQ\bar{U}\bar{E}, \bar{H}^*, Q\bar{U}L^*, \bar{U}\bar{D}^*\bar{E}) = -(2l + e) \quad (65)$$

$$Q_X(QQQ\bar{L}, \bar{U}\bar{U}\bar{D}\bar{E}) = 0 \quad (66)$$

For the terms in Eq(64,65) the discussion is identical to that of the dimension four terms above. They are suppressed by ϵ^7 in any effective theory. The last two terms are $B - L$ invariant and they can not be excluded by any gauged $U(1)_X$, given our assumptions. It is however not so clear whether these terms actually pose a problem. In Ref.[2] the partial proton decay rate via the operator $(QQQL)_F$ was estimated to be

$$\Gamma_p(QQQL) \approx \left(\frac{k^2 e^2}{8\pi^2} \right)^2 \frac{m_{proton}^5}{(M\tilde{m})^2}. \quad (67)$$

Here M is the mass suppression of the non-renormalizable dimension-5 term in the effective Lagrangian. For us $M = \kappa^{-1}$. e is the electric coupling which enters when the final state s-fermions are converted via electroweak gaugino exchange to their R-parity even partners. This involves a 1-loop diagram, hence the $8\pi^2$. \tilde{m} is an effective supersymmetric mass, *e.g.* $\tilde{m} = m_{gaugino}/m_{squark}^2$. k^2 is the coupling constant of the effective operator; it is squared since when generated from renormalizable interactions it involves at least two coupling constants. The dominant decay mode corresponding to (67) is $p \rightarrow K^+ \nu$ [4]; the experimental bound is [19] $\tau_{proton} > 10^{32}a$. This then corresponds to

$$k < 10^{-4} \left(\frac{\tilde{m}}{100 GeV} \right)^{1/2}. \quad (68)$$

This is the same order as the muon Yukawa coupling in the Standard Model, which is not explained but is generally also not considered to be unnatural. None the less, we proceed to consider the conditions where these terms are suppressed. In order for the terms (66) to be induced at the loop level when $U(1)_X$ is broken, they must occur through the exchange of Z_i superfields. This implies these terms are of the form

$$f(Z_1, Z_2, Z_3)(\alpha QQQL + \beta \bar{U}\bar{U}\bar{D}\bar{E}) \quad (69)$$

α, β are constants and $f(Z_1, Z_2, Z_3)$ must have $U(1)_X$ charge zero. The lowest order contribution is for $f(Z_1, Z_2, Z_3)$ to be of the same form as g' in (40)

$$f(Z_1, Z_2, Z_3) = \kappa^{10} Z_3^4 \sum_{p=0}^5 Z_1^{5-p} Z_2^p \quad (70)$$

and this gives rise to the effective coupling

$$G_5 = \langle f \rangle \sim \kappa^{10} \langle Z_i \rangle^9 \sim 10^{-36} GeV^{-1} \quad (71)$$

which is highly suppressed. A constant term corresponds to a tree-level term and is not allowed. Thus the dimension five terms are safe as well. However, when the unknown unified theory is broken, and before $U(1)_X$ is broken the $QQQL$ term could be generated directly in the effective theory. In this case the coupling must satisfy the above bound (68).

5 LSP Lifetime and Neutrino Masses

We now turn to two applications of our analysis. We have found that in general we expect an effective R -parity breaking superpotential (58) with highly suppressed but non-zero couplings

$$\lambda \approx \lambda' \approx \lambda'' \approx 10^{-21}. \quad (72)$$

The terms of $g^{eff}(S_i)$ induce the decay of the lightest supersymmetric particle (LSP).⁵ For a photino LSP the decay rate has been calculated by Dawson [20]

$$\Gamma_{\tilde{\gamma}} = \frac{\alpha \lambda^2}{128 \pi^2} \frac{m_{\tilde{\gamma}}^5}{M_{\tilde{f}}^4}. \quad (73)$$

α is the fine structure constant, λ is the coupling in g^{eff} and $M_{\tilde{f}}$ is for example the scalar electron mass. The more general decay of a neutralino LSP is given in [10]. The lifetime of the LSP is then given in natural units as

$$\tau_{\tilde{\gamma}} = 10^{23} s \left(\frac{10^{-21}}{\lambda} \right)^2 \left(\frac{50 GeV}{m_{\tilde{\gamma}}} \right)^5 \left(\frac{M_{\tilde{f}}}{150 GeV} \right)^4. \quad (74)$$

The large difference in lifetime compared to the proton is due to the different masses and the two powers less dependence on λ

$$\frac{\tau_{\tilde{\gamma}}}{\tau_{proton}} \approx \left(\frac{m_{lsp}}{m_{proton}} \right)^5 \left(\frac{e}{\lambda} \right)^2 \approx 10^{49} \quad (75)$$

for $m_{\tilde{\gamma}} = 50 GeV$; e is the electric charge quantum.

If we allow a supersymmetric mass range up to $1 TeV$ for the scalar fermion mass and LSP masses up to $M_{\tilde{f}}$ we obtain in terms of the lifetime of the universe $\tau_u = 2 \cdot 10^{17} s$

$$10^3 < \frac{\tau_{\tilde{\gamma}}}{\tau_u} < 10^7. \quad (76)$$

There are several large uncertainties in this result. First, we have taken the singlet vev $\langle Z \rangle = \mathcal{O}(10^{16} GeV)$. The insertion of a simple factor of three either way changes $\lambda = \epsilon^7$ by a factor $10^{\pm 3}$ and $\tau_{\tilde{\gamma}}$ by a factor $10^{\pm 6}$! Second, we have only considered a photino LSP. As discussed in Ref.[10] in detail the LSP lifetime can vary by many orders of magnitude for a general neutralino LSP, depending on the MSSM parameters.

Surprisingly enough there are severe constraints on a long-lived but unstable LSP [21, 22], even for $\tau_{\tilde{\gamma}} > \tau_u$. The relic density $\Omega_{\tilde{\gamma}} h^2$ of a stable LSP is typically in the range $10^{-3} - 10$ [23]; if constrained to the *minimal* $N = 1$ local supersymmetric model it is $10^{-2} - 10$ [24]. For $\tau_{\tilde{\gamma}}$ as large as $10^7 \tau_u$ only one in 10^7 LSP will have decayed today. However, for such large relic densities this is still a large number and the decay products can lead to observable effects. In particular final-state decay neutrinos can be observed in the large underground detectors. For a LSP mass below

⁵The allowed dimension 5 terms of (66) conserve R -parity and are thus irrelevant for the decay of the LSP.

1 TeV the strictest bound [21] comes from the experimental upper limit on upward-going muons from the IMB detector [25]. In terms of the LSP branching ratio B_{ν_μ} to muon neutrinos this bound can be expressed as [21]

$$\Omega_{\tilde{\gamma}} h^2 < 4 \cdot 10^{-9} B_{\nu_\mu}^{-1} \frac{\tau_{\tilde{\gamma}}}{\tau_u} \ln^{-1} \left(1 + \frac{m_{\tilde{\gamma}}}{1.5 TeV} \right) \quad (77)$$

Assuming all operators in (58) have equal strength the muon-neutrino branching fraction is of order $1/5$. The lifetime is reduced by about a factor of 35, assuming $m_{\tilde{\gamma}} < m_{top}$.⁶ For an LSP mass of 150 GeV we thus obtain

$$\Omega_{\tilde{\gamma}} h^2 < 6 \cdot 10^{-9} \frac{\tau_{\tilde{\gamma}}}{\tau_u}, \quad (78)$$

and we see that only the upper range in Eq.(76) is still allowed. This analysis is a bit crude in many respects. For example, both the relic density and the LSP lifetime depend strongly on the MSSM parameters and it would be appropriate to perform a correlated analysis. This is beyond the scope of this paper.

Thus although the LSP lifetime is many orders of magnitude shorter than the experimental bound on the proton lifetime ($\tau_{proton} > 10^{22} \tau_u$) the resulting bound on the couplings $\lambda, \lambda', \lambda''$ of (58) is actually 2 – 3 orders of magnitude stricter, if all operators are of equal strength.

The non-renormalizable interactions (56) can also give rise to neutrino masses. The singlets Z_i can couple to the left-handed neutrinos via

$$\kappa^6 L \bar{H} \prod_i Z_i \quad (79)$$

leading to Dirac neutrino masses of order $\epsilon^6 < \bar{H} >$. The leading Majorana neutrino mass for the singlets is given in (fermionic) analogy to Eq.(45); it is of order $m_{3/2} = \mathcal{O}(100 GeV)$. We then obtain the see-saw mass matrix

$$\begin{pmatrix} 0 & \epsilon^6 < \bar{H} > \\ \epsilon^6 < \bar{H} > & m_{3/2} \end{pmatrix}. \quad (80)$$

The light-neutrino mass is given by

$$m_\nu \approx \frac{(\epsilon^6 < \bar{H} >)^2}{m_{3/2}} \approx 3 \cdot 10^{-18} eV, \quad (81)$$

which is extremely small and unobservable. This is consistent with all experiments but can not explain the hints for neutrino masses such as the solar neutrino puzzle.

6 Conclusion

We have found the most general anomaly-free $U(1)_X$ with three G_{SM} singlets. In local supersymmetry the breaking of $U(1)_X$ is predicted to be at the scale $10^{16} GeV$.

⁶For $m_{\tilde{\gamma}} > m_{top}$ these numbers change to $1/6$ and 45 ; the product only changes from 7 to 7.5 .

The most general gauge-invariant non-renormalizable superpotential is consistent with proton decay experiments. It is only barely consistent with the cosmological bounds on the decay of the lightest supersymmetric particle. This poses a severe constraint for all supersymmetric models.

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